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NONSTEADY METHOD OF DETERMINING HEAT FLUX

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A method is described for determining intense heat flux, based on solution of the linearized heat-conduction equations. The data obtained are compared with data determined by the steady calorimeter method and the quasisteady method.

In present practice intense heat flux is measured by a number of techniques which have certain defects as well as advantages.

For example, the steady types of calorimeters are typically of complex construction and have definite cooling limitations in large heat-flux conditions, i.e., it is impossible to eliminate the maximum heat rate in a short time interval (because of the properties of the calorimeter material). Therefore, nonsteady methods of measuring intense heat flux have been developed recently. In particular, it was proposed in [1] to measure heat flux using a sensor which is so short that the temperature difference between the front and back walls would be negligible. This assumption will be valid only for a thin-walled sensor [2]. Otherwise, measurement of a large heat flux can introduce considerable error. For a thin sensor the measurement of heat flux requires the use of high-speed recording equipment and materials to withstand a large heat load, because of the absence of heat removal.

In [3] it has been suggested to measure heat flux by the use of quasistationary heat conditions. It should be noted that the assumption of equality of heating rates on the forward and rear sensor walls is based on solution of the linear heat-conduction equation. This assumption breaks down if nonlinearity is taken into account. Experiments have also confirmed that the rate of heating of a body differs at each point. Therefore, first of all, this method is based on an *a priori* incorrect assumption and, therefore, contains an *a priori* inherent error. Secondly, for the formula $q = \sigma\rho C(dt/d\tau)$, on which the theory of this method is based, to apply, it is necessary that the temperatures and the heating rates be the same throughout the entire body. But if we assume that the heating rates are equal at all points of the body, as the quasistationary method suggests, the temperatures at the forward and rear walls will be quite different. Therefore, at these points there will also be different values of specific heat capacity, and this must be accounted for in determining the heat flux. However, this difference cannot be accounted for in the above equation. Therefore, there is an additional error in the method.

Several papers have proposed to determine heat flux using sensors containing several thermocouples. For example, a method was proposed in [4-6] for measuring heat flux using sensors containing four thermocouples. This method differs from those described above in

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that it takes into account the nonlinearity of the heat-conduction equation. However, this method requires a large volume of computation on a high-speed computer.

Another method differing somewhat from those proposed above was suggested in [7], the object being to reduce the number of included thermocouples: the method is based on solution of the nonlinear heat-conduction equation, linearized to a high degree of accuracy.

For example, for a sensor with two thermocouples, one close to the heated surface and the other on the opposite surface, to determine the desired heat flux one must solve the following system of equations:

$$\frac{\partial^2\Theta_3}{\partial\tau^2} = a_0 \frac{\partial^3\Theta_3}{\partial x^2\partial\tau} \ (R_1 < x < R_2,\ \tau > 0), \tag{1}$$

$$\Theta_3|_{\tau=0} = 0 \ (R_1 < x < R_2), \tag{2}$$

$$\left.\frac{\partial\Theta_3}{\partial\tau}\right|_{\tau=0} = 0, \tag{3}$$

$$\Theta_3|_{x=R_1} = 2\varphi_{R_1}(\tau) + \frac{1}{2}\left(\frac{C_1}{C_0} + \frac{\lambda_1}{\lambda_0}\right)\varphi^2_{R_1}(\tau) = \psi_{R_1}(\tau), \tag{4}$$

$$\Theta_3|_{x=R_2} = 2\varphi_{R_2}(\tau) + \frac{1}{2}\left(\frac{C_1}{C_0} + \frac{\lambda_1}{\lambda_0}\right)\varphi^2_{R_2}(\tau) = \psi_{R_2}(\tau), \tag{5}$$

where

$$\Theta_3(x,\ \tau) = 2\Theta(x,\ \tau) + \frac{1}{2}\left(\frac{C_1}{C_0} + \frac{\lambda_1}{\lambda_0}\right)\Theta^2(x,\ \tau);\ \Theta(x,\ \tau) = t(x,\ \tau) - t_0; \tag{6}$$

a_0 is the thermal diffusivity, at the initial temperature t_0; $\varphi_{R_1}(\tau)$ and $\varphi_{R_2}(\tau)$ are the temperature fields determined experimentally at the points $x = R_1$ and $x = R_2$, respectively; λ_0, λ_1, C_0, and C_1 are the coefficients of the equations

$$\lambda(\Theta) = \lambda_0 + \lambda_1\Theta, \tag{7}$$

$$C(\Theta) = C_0 + C_1\Theta. \tag{8}$$

Restricting ourselves to derivatives of the functions $\psi_{R_1}(\tau)$ and $\psi_{R_2}(\tau)$ of only the first order, the solution of the system of equations (1)-(5) has the form

$$\begin{aligned}\Theta_3(x,\ \tau) &= \psi_{R_1}(\tau)\frac{R_2 - x}{R_2 - R_1} + \psi_{R_2}(\tau)\frac{x - R_1}{R_2 - R_1} + \\ &+ \psi'_{R_1}(\tau)\frac{(R_2 - x)^3 - (R_2 - R_1)^2(R_2 - x)}{6a_0(R_2 - R_1)} + \\ &+ \psi'_{R_2}(\tau)\frac{(x - R_1)^3 - (R_2 - R_1)^2(x - R_1)}{6a_0(R_2 - R_1)},\end{aligned} \tag{9}$$

and Fourier's law gives the following formula for the desired heat flux:

$$\begin{aligned}q = &-[\lambda_0 + \lambda_1(t(x,\ \tau) - t_0)] \times \\ &\times \left[4 + 2\left(\frac{C_1}{C_0} + \frac{\lambda_1}{\lambda_0}\right)\Theta_3(x,\ \tau)|_{x=0}\right]^{-\frac{1}{2}} \left.\frac{\partial\Theta_3(x,\ \tau)}{\partial x}\right|_{x=0}.\end{aligned} \tag{10}$$

This method has a number of advantages over the methods described above. First, it has high accuracy, because it allows for nonlinear terms in the heat-conduction equation. Secondly, the solutions of Eq. (9) and formula (10) are so simple that one need only use very simple computational methods. Finally to determine the desired heat flux it is enough to have only two measurement points in the sensor, which simplifies the experiment. In addition, the method permits determination of heat flux throughout the whole time of operation of the heat source.

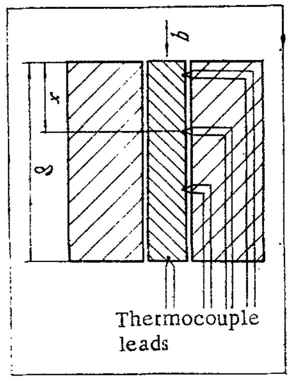

Fig. 1. Calorimetric sensor for determining heat flux.

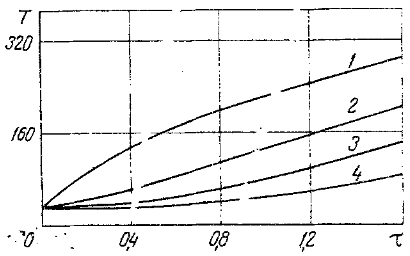

Fig. 2

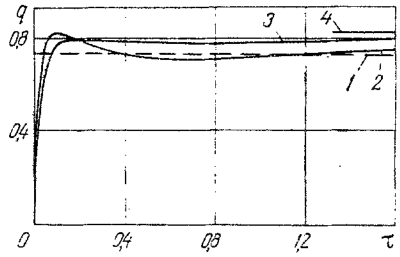

Fig. 3

Fig. 2. Heating characteristics of the calorimeter at the points x = 2 (1), 8 (2), 13 (3), and 19.5 mm (4).

Fig. 3. Results of computed heat flux: 1) steady-state calorimeter; 2) using Eq. (10); 3) using Eq. (17); 4) using the quasisteady technique.

It should be noted that without exception all of the techniques described require knowledge of the temperature dependence of the thermophysical characteristics of the sensor material, in order to increase the accuracy of heat-flux determination. However, it is often assumed in practice that the temperature dependence of the thermophysical characteristics of the sensor material is unknown. Nevertheless, the heat flux must be determined.

We now examine this question. As a basis for our theory we will take the linear heat-conduction equations of order greater than the second, as described in [8]. For example, if the sensor body contains three thermocouples, one must solve a system of equations of the following type:

$$\frac{\partial^2\Theta}{\partial x\partial\tau}=a_0\frac{\partial^3\Theta}{\partial x^3}\quad(R_1<x<R_3,\ \tau>0), \tag{11}$$

$$\Theta|_{\tau=0}=0\quad(R_1<x<R_3), \tag{12}$$

$$\Theta|_{x=R_1}=\varphi_1(\tau)\quad(\tau>0), \tag{13}$$

$$\Theta|_{x=R_2}=\varphi_2(\tau)\quad(\tau>0), \tag{14}$$

$$\Theta|_{x=R_3}=\varphi_3(\tau)\quad(\tau>0), \tag{15}$$

where $\Theta(x, \tau) = t(x, \tau) - t_0$; a_0 is the thermal diffusivity of the sensor material, at temperature t_0; $\varphi_1(\tau)$, $\varphi_2(\tau)$, and $\varphi_3(\tau)$ are the temperatures as functions of time, determined experimentally at the points $x = R_1$, $x = R_2$, and $x = R_3$, respectively.

Since the system of equations (11)-(15) is linear, there are no difficulties in the solution. It is convenient here to use Laplace transforms, which lead to the following solution:

$$t(x,\ \tau)=t_0+\varphi_1(\tau)+[\varphi_1(\tau)-\varphi_2(\tau)]\frac{(R_3-x)(x-R_1)}{(R_3-R_2)(R_1-R_2)}+[\varphi_1(\tau)-\varphi_3(\tau)]\frac{(x-R_1)(R_2-x)}{(R_3-R_2)(R_3-R_1)}+$$

$$+[\varphi_1'(\tau)-\varphi_2'(\tau)]\frac{(R_3-x)(x-R_1)[(x-R_1)^2+(R_3-x)^2-(R_1-R_2)^2-(R_3-R_2)^2]}{24a_0(R_3-R_2)(R_1-R_2)}$$

$$+[\varphi_1'(\tau)-\varphi_3'(\tau)]\,\frac{(x-R_1)(R_2-x)[(x-R_1)^2+(R_2-x)^2-(R_3-R_2)^2-(R_3-R_1)^2]}{24a_0(R_3-R_2)(R_3-R_1)}. \tag{16}$$

Here, as was done in the solution of Eq. (9), we can restrict ourselves to derivatives of the functions $\varphi_1(\tau)$, $\varphi_2(\tau)$, and $\varphi_3(\tau)$ of first order. This restriction is valid in that any of the curves $\varphi_1(\tau)$, $\varphi_2(\tau)$, and $\varphi_3(\tau)$ can be approximated by a straight line in a small time interval $\Delta\tau$. In this case the heat flux is determined, as usual, by the formula

$$q=-\lambda(t)\frac{\partial t}{\partial x}\bigg|_{x=0}. \tag{17}$$

To verify solution (16) and Eq. (17), we carried out an experiment (Fig. 1). A copper calorimetric sensor of length 20 mm and diameter 6 mm was mounted at a specific distance from the end of the nozzle of an electric arc air heater, located in a thermally insulated bushing around its forward end surface. Four thermocouples were included along the sensor, at distances 2, 8, 13, and 19.5 mm from the coordinate origin. The temperatures curves taken with a type N-700 oscilloscope at these points are shown in Fig. 2, while Fig. 3, curve 3 shows the variation in heat flux as a function of time. Curve 2 of Fig. 3 shows the variation of the heat-flux values determined by means of Eq. (10).

Experiments were carried out analogously with a water-cooled calorimeter. The heat flux obtained from these experiments is shown by curve 1. If we consider the readings of curve 1 to be the most probable, as is assumed in experimental investigations, then Fig. 3 shows that the maximum deviation of curves 2 and 3 from curve 1 in the time interval $0<\tau<0.5$ is on the order of 7%, which can be considered a satisfactory agreement. Finally, curve 4 shows the heat flux determined by the quasisteady method [3]. As was expected, the deviation of curves 1 and 4 are the largest, on the order of 14%.

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